1 – LIMITS OF THE RIGID METHOD
Under the hypothesis of Limit Equilibrium, the slope stability analysis considers the possible reinforcements as concentrated forces situated along the considered surface, at the intersection between the reinforcement and the surface itself. The module of these forces is taken as the minimum between the breaking load of the reinforcement and the pullout one, each one reduced through an appropriate safety factor that depends on the view standard.

This approach is based on the implicit hypothesis that the reinforcements, in the reinforced soil structure under analysis, always work in a synchronic way, whatever is their geometric configuration and the related flexible characteristics. That hypothesis is surely convenient because simplifies a lot the computation procedure, but from a mathematical point of view it is coarse and not protective.

In fact, to assume the synchronous loading of the reinforcements located along the given surface, means to take into account the configuration which maximise the stabilising forces given by the reinforcements.

A more restrictive approach points out that the different stiffness of the reinforcements imply a not uniform distribution of the forces, which are taken at first by the stiffer reinforcements and then by the weakest ones. In technical literature a computation method called “Displacement method” is available, which can evaluate the different performance of the reinforcements, caused by different stiffness even in conditions of rigid equilibrium.

2 – DESCRIPTION OF THE DISPLACEMENTS METHOD
Therefore the ‘Displacement method’ is not a calculation method to evaluate the slope stability; it is a procedure used to define the strength pattern (along the considered sliding surface) which simulates the presence of reinforcements. The stability analysis is made in rigid equilibrium conditions so that even the classical formulations (Bishop, Janbu, etc.) remain unchanged. The limit of the ‘Displacement method’ is the need of circular sliding surfaces. In order to evaluate the forces used to simulate the presence of reinforcements during the stability analysis the procedure is:

Once the sliding surface for which the safety factor has to be found, you have to suppose a rigid rotation of the part of the slope given by the surface itself and from the external perimeter of the slope. This rotation can produce in each point of the sliding surface a \( \delta \) movement related to the rotation itself and to the geometric characteristics of the surface through the relation:

\[
\delta = R \cdot d\alpha
\]

where \( R \) is the radius of the sliding surface and \( d\alpha \) is the rotation angle. If the movements are very little, and this is acceptable for slope stability analysis the \( \delta \) movement can be taken as straight.

Moreover if you think that this movement corresponds to the elongation of the reinforcement located in a tangential direction with respect to the surface itself, only the horizontal component \( \delta_x \) is the cause of the reacting forces.

Once \( \delta \) is known it is possible to calculate the effective component in order to determine the efforts of the reinforcement through the relation:

\[
\delta_{xi} = \delta \cdot \cos \theta_i
\]

\( \theta_i \) is the angle between the i-th reinforcement and the tangent line to the sliding surface at the intersection with the reinforcement.

Once determined \( \delta_{xi} \) for all the reinforcements found on the sliding surface, known the linear stiffness of each reinforcement, it is possible to determine the force \( f_i \) from the following relation:

\[
f_i = K_i \cdot \delta_{xi}
\]

where \( K_i \) is the linear stiffness of the i-th reinforcement.
The above mentioned relation suggests an indefinite elastic behaviour of the reinforcement material. Practically each reinforcement has a breaking load and, moreover, before achieving it, the pullout load could be gained. Therefore each reinforcement will be simulated with a force whose value is the minimum between the breaking load and the pullout load of the i-th reinforcement.

Practically each reinforcement is simulated through a force whose intensity is equal to the minimum between the Pullout load and the one generated by the $\delta$ movement. When the load created by the displacement is equal or superior to the breaking load of the reinforcement, we may have:

- If the reinforcement is scarcely anchored, the cinematic mechanism causes its pullout and the reinforcement will be simulated with its force Pullout.
- If the reinforcement is well anchored, the cinematic mechanism breaks the reinforcement which will be simulated by a force equal to zero.

In reality what we have described above is only valid for reinforcements with elasto-fragile behaviour. In the implementation of the Displacement method a control to verify also a plastic range which exceeds the reinforcement resistance has been made. The elasto-fragile or elasto-plastic characteristics are peculiarity of each reinforcement and the relevant information are in the general data base of the reinforcements.

3 – INFLUENCE OF THE DISPLACEMENT ON THE SLOPE STABILITY FACTOR

The stability analysis, made simulating the reinforcement according to the model proposed by the ‘Displacement method’, leads to results very different from the ones given by the traditional approach. Let’s see this influence and compare the stability analysis of the Displacement method with the traditional one.

While the traditional method leads to a Safety Factor (FS) which depends on the examined geometry and on the geotechnical parameter, the Displacement method leads to an FS value which depends also on the displacement value ($\delta$). As a consequence, for an assigned geometry, each analysis surface has only one FS value with the traditional method and infinite FS values following the ‘Displacement method’. As for the link between FS and the $\delta$ displacement, this is a little bit complex and depends on parameters like the slope geometry, the soil geotechnical parameters and those relevant to soil-reinforcement interaction, reinforcement distribution, reinforcements’ deformative characteristic, etc. However there are some very important elements, which are common to any kind of geometry. Considering a reinforced slope and a generic sliding surface with a certain number of reinforcements, the curve $FS=FS(\delta)$ Has got a qualitative trend as the one of the following picture:
For $\delta=0$ the FS value corresponds to the one valuable with traditional rigid approach for the same slope when there are no reinforcements. For each analysis configuration there is always a particular displacement value ($\delta_u$) which corresponds to the displacement that causes a breakage in all the reinforcements, beyond which the FS value remain constant and equal to the one correspondent to $\delta=0$.

For $\delta$ values included in the range $0-\delta_u$ the FS($\delta$) function may be different but in any case there will be particular singular points (discontinuities) in correspondence of $\delta$ values which produce breakages in some reinforcements.

4 – THE DISPLACEMENTS METHOD IMPLEMENTED IN MACSTARS W

4.1 ADDITIONAL REQUIRED DATA ON THE REINFORCEMENTS

The stability analysis of a reinforced slope with the Displacement method requires, differently from the traditional method, some more input details: the elastic characteristics of the reinforcement, the parameters related to the soil-reinforcement interaction model and the value of the displacement to evaluate the force pattern used to simulate the reinforcements. These data, apart from the displacement value, are intrinsic reinforcements’ property so they have been input in the general data base of the reinforcements. The displacement is variable and it depends on the geometry of the analysis surface and on the check conditions fixed by the designer.
4.2 ELASTIC AND PLASTIC LIMIT METHOD
The variability of the 'Displacement' parameter both with the geometry and with the user's check conditions, has brought to implement two different options (apart from the Rigid Method):

- Limit ELASTIC deformation method
- Limit PLASTIC deformation method

ELASTIC LIMIT
For each analysed surface, the assigned displacement is increased from 0 up to the maximum value at which no reinforcement is plasticized. In other words, the increasing displacement is stopped as soon as one reinforcement reaches its own elastic limit, after which the reinforcement would break (if elastic) or plasticize (if elasto-plastic).

For each surface Macstars selects the maximum safety factor; among all the analysed surface the one corresponding to the minimum safety factor is finally selected.

PLASTIC LIMIT
For each analysed surface, the assigned displacement is increased from 0 up to the maximum value at which no reinforcement is broken (even though it can be plasticized).
For each surface Macstars selects the maximum safety factor; among all the analysed surface the one corresponding to the minimum safety factor is finally selected.

At the end of the calculation process the following outputs are given:
- SF of reinforced slope equal to the minimum of all the examined FS(δ) functions;
- The value of the maximum SF of the drawn surfaces;
- The displacement corresponding to SF min;
- The forces in the reinforcement for SF max;
- The forces in the reinforcement for SF=user assigned value (in this case equal to 1.3);
- The forces in the reinforcement for SF=1

Output of the Plastic limit method

Obviously the value of SF obtained from the Elastic limit method cannot be greater than SF calculated from the Plastic limit method: the 2 values can coincide only if all the reinforcements in the structure are elastic.

The Plastic Limit option, much heavier from a calculation point of view than the Elastic Limit one, allows to obtain much more information about the slope stability and gives indications about the situation of the loads in the reinforcement. In input the program requires to specify, apart from the general data for the stability analysis, the required Safety Factor for which to know the stress situation is needed.

4.3 ASSIGNED DISPLACEMENT METHOD

Once the critical surface has been obtained (with the Elastic or Plastic limit method) it is possible to run thorough analysis of the structure’s behaviour: after having input the coordinates of the critical surfaces as Assigned Surface check, by varying the assigned displacement the whole tensile-elongation reinforcements behaviour can be determined.

5 – DIFFERENCES BETWEEN THE SF OBTAINED WITH THE DIFFERENT METHODS

From a quantitative point of view the difference between the FS value obtained with a traditional approach and the FS(δ) maximum value in the 0–δ, range calculated through the Displacement method is very evident. In particular the FS value calculated with the traditional method is always higher because, as it assumes that the reinforcements are synchronically working, includes, in the solving equation used to calculate FS, resistance resources that in reality are not available.

The following table shows the numerical results of the slope stability factor with the different methods obtained on various structure typologies.
<table>
<thead>
<tr>
<th>CASE</th>
<th>SF RIGID</th>
<th>SF ELASTIC</th>
<th>Δ %</th>
<th>SF PLASTIC</th>
<th>Δ %</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.470</td>
<td>1.459</td>
<td>-0.75</td>
<td>1.470</td>
<td>0.00</td>
<td>Paralink block over Terramesh block, structural embankment’s phi=30°</td>
</tr>
<tr>
<td>1 bis</td>
<td>1.184</td>
<td>1.078</td>
<td>-8.95</td>
<td>1.141</td>
<td>-3.63</td>
<td>Like 1 but phi=20°</td>
</tr>
<tr>
<td>2</td>
<td>1.314</td>
<td>1.256</td>
<td>-4.41</td>
<td>1.291</td>
<td>-1.75</td>
<td>Mixed structure: Terramesh+Paragrid phi=35°</td>
</tr>
<tr>
<td>2 bis</td>
<td>0.870</td>
<td>0.816</td>
<td>-6.21</td>
<td>0.852</td>
<td>-2.07</td>
<td>Like 2 but phi=25°</td>
</tr>
<tr>
<td>3</td>
<td>1.399</td>
<td>1.347</td>
<td>-3.72</td>
<td>1.374</td>
<td>-1.79</td>
<td>Bi-facial: only Terramesh</td>
</tr>
<tr>
<td>4</td>
<td>2.379</td>
<td>1.614</td>
<td>-32.16</td>
<td>2.069</td>
<td>-13.03</td>
<td>Bi-facial: Terramesh+Paralink, phi=35°</td>
</tr>
<tr>
<td>4 bis</td>
<td>1.350</td>
<td>1.092</td>
<td>-19.11</td>
<td>1.268</td>
<td>-6.07</td>
<td>Like 4 but phi=25°</td>
</tr>
<tr>
<td>4 ter</td>
<td>1.350</td>
<td>1.274</td>
<td>-5.63</td>
<td>1.274</td>
<td>-5.63</td>
<td>Like 4-bis, but Paralink only</td>
</tr>
<tr>
<td>5</td>
<td>1.629</td>
<td>1.602</td>
<td>-1.66</td>
<td>1.602</td>
<td>-1.66</td>
<td>Bi-facial: Terramesh+Paralink, phi=38°</td>
</tr>
<tr>
<td>5 bis</td>
<td>1.106</td>
<td>1.093</td>
<td>-1.18</td>
<td>1.093</td>
<td>-1.18</td>
<td>Like 5, phi=28°</td>
</tr>
<tr>
<td>6</td>
<td>1.045</td>
<td>1.005</td>
<td>-3.83</td>
<td>1.024</td>
<td>-2.01</td>
<td>Bi-facial Terramesh only, phi=28°</td>
</tr>
<tr>
<td>6 bis</td>
<td>1.677</td>
<td>1.580</td>
<td>-5.78</td>
<td>1.637</td>
<td>-2.39</td>
<td>Like 6, phi=38°</td>
</tr>
<tr>
<td>7</td>
<td>1.929</td>
<td>1.581</td>
<td>-18.04</td>
<td>1.800</td>
<td>-6.69</td>
<td>Terramesh+Paralink</td>
</tr>
<tr>
<td>7 bis</td>
<td>1.431</td>
<td>1.369</td>
<td>-4.33</td>
<td>1.431</td>
<td>0.00</td>
<td>Like 7, Terramesh only</td>
</tr>
</tbody>
</table>

**Diagram:**

Case 1: Paralink block over Terramesh block, structural embankment with phi=30°
Case 1-bis: like Case 1 but \( \phi = 20^\circ \)

Case 2: mixed structure alternating Terramesh and ParaGrid, \( \phi = 35^\circ \)
Case 2-bis: like Case 2, $\phi=25^\circ$

Case 3: Terramesh only

### Tipo di Rinforzo

<table>
<thead>
<tr>
<th>Case</th>
<th>L1</th>
<th>H1</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>16</td>
<td>1</td>
<td>16.6</td>
</tr>
<tr>
<td>B2</td>
<td>15</td>
<td>1</td>
<td>16.6</td>
</tr>
<tr>
<td>B3</td>
<td>11</td>
<td>1</td>
<td>16.6</td>
</tr>
<tr>
<td>B4</td>
<td>1</td>
<td>0.61</td>
<td>20</td>
</tr>
<tr>
<td>B5</td>
<td>0.61</td>
<td>7.32</td>
<td>20</td>
</tr>
</tbody>
</table>

Macromesh Terramesh 602 FP - 1000
Terram PaveGrid 2015

### Tipo di Rinforzo

<table>
<thead>
<tr>
<th>Case</th>
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<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM_1</td>
<td>17.65</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Macromesh Terramesh 602 FP - 1000
Terram PaveGrid 2015

FS = 0.816

FS = 1.374
### Case 4: bi-facial Terramesh and Paralink structure, phi=35°

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Design Parameters</th>
<th>Reinforcement Type</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>L=10.00, W=2.00, phi=35°</td>
<td>Terrameb, F=400,000</td>
<td>1.614</td>
</tr>
<tr>
<td>B2</td>
<td>L=8.00, W=1.74, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>1.74</td>
</tr>
<tr>
<td>B3</td>
<td>L=21.40, W=2.32, phi=35°</td>
<td>Terrameb, F=400,000</td>
<td>1.32</td>
</tr>
<tr>
<td>B4</td>
<td>L=18.00, W=4.06, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>2.06</td>
</tr>
<tr>
<td>B5</td>
<td>L=12.00, W=3.48, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>3.48</td>
</tr>
</tbody>
</table>

### Case 4-bis: like case 4 but phi=25°

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Design Parameters</th>
<th>Reinforcement Type</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>L=10.00, W=2.00, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>1.092</td>
</tr>
<tr>
<td>B2</td>
<td>L=9.00, W=1.74, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>1.74</td>
</tr>
<tr>
<td>B3</td>
<td>L=21.40, W=2.32, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>1.32</td>
</tr>
<tr>
<td>B4</td>
<td>L=18.00, W=4.06, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>2.06</td>
</tr>
<tr>
<td>B5</td>
<td>L=12.00, W=3.48, phi=25°</td>
<td>Terrameb, F=400,000</td>
<td>3.48</td>
</tr>
</tbody>
</table>
Case 4-ter: like 4-bis ($\phi=25^\circ$) but Paralink only

Case 5: bi-facial Terramesh+Paralink structure, $\phi=38^\circ$
Case 5-bis: like case 5 but \( \phi = 28^\circ \)

<table>
<thead>
<tr>
<th>Case</th>
<th>( L )</th>
<th>( H )</th>
<th>( \phi )</th>
<th>Reinforcement Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>90.00</td>
<td>5.00</td>
<td>10.0</td>
<td>Macstar Terramesh System 62 TP - 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B2</td>
<td>62.00</td>
<td>5.00</td>
<td>10.0</td>
<td>Macstar Terramesh System 62 TP - 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B3</td>
<td>74.00</td>
<td>5.22</td>
<td>25.0</td>
<td>Macstar Terramesh System 66 ST TP - 0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B4</td>
<td>63.00</td>
<td>5.22</td>
<td>25.0</td>
<td>Macstar Terramesh System 65 ST TP - 0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B5</td>
<td>52.00</td>
<td>5.22</td>
<td>25.0</td>
<td>Macstar Terramesh System 65 ST TP - 0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
</tbody>
</table>

Case 6: bi-facial Terramesh only structure, \( \phi = 28^\circ \)

<table>
<thead>
<tr>
<th>Case</th>
<th>( L )</th>
<th>( H )</th>
<th>( \phi )</th>
<th>Reinforcement Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>60.00</td>
<td>5.00</td>
<td>10.0</td>
<td>Macstar Terramesh System 62 TP - 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B2</td>
<td>51.00</td>
<td>5.00</td>
<td>10.0</td>
<td>Macstar Terramesh System 62 TP - 0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
<tr>
<td>B3</td>
<td>44.00</td>
<td>5.22</td>
<td>25.0</td>
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<td></td>
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<td>Terramesh 200M</td>
</tr>
<tr>
<td>B4</td>
<td>32.00</td>
<td>5.22</td>
<td>25.0</td>
<td>Macstar Terramesh System 65 ST TP - 0.36</td>
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<tr>
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<td>21.00</td>
<td>5.22</td>
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<td></td>
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<td></td>
<td></td>
<td>Terramesh 200M</td>
</tr>
</tbody>
</table>
Case 6-bis: like case 6 but \( \phi = 38^\circ \)

Case 7: Terramesh+Paralink
6 – PRACTICAL EXAMPLE ON HOW TO USE THE DISPLACEMENTS METHOD

For a better understanding of the previous chapters, a complete calculation is reported hereafter: the structure is a mixed one (with both elastic and elasto-plastic reinforcements, i.e. respectively Paralink and Terramesh) and the checks have been carried out with the 3 available methods: Rigid, Elastic and Plastic limit, running the block 1 internal stability.

Furthermore, the critical surface obtained from the Plastic method has been analysed with the Assigned Displacement method by imposing increasing values of the displacement.
Rigid method: SF=1.350

Elastic Limit method: SF=1.087
Plastic limit method: SF = 1.268

The results are shown in the following table:

<table>
<thead>
<tr>
<th>FS RIGIDO</th>
<th>FS ELASTICO</th>
<th>Differenza %</th>
<th>FS PLASTICO</th>
<th>Differenza %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.350</td>
<td>1.087</td>
<td>-19.48</td>
<td>1.268</td>
<td>-6.07</td>
</tr>
</tbody>
</table>

Variability of SF vs. the assigned displacement
<table>
<thead>
<tr>
<th>METODO</th>
<th>SPOSTAMENTO (m)</th>
<th>FS</th>
<th>Blocco B1</th>
<th>Blocco B4</th>
<th>Blocco B5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y=1.16</td>
<td>Y=2.32</td>
<td>Y=0</td>
<td>Y=1.74</td>
<td>Y=3.48</td>
</tr>
<tr>
<td>RIGIDO</td>
<td>0</td>
<td>1.356</td>
<td>0.561</td>
<td>0.361</td>
<td>0.138</td>
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<tr>
<td>ELASTICO</td>
<td>0.0762</td>
<td>1.088</td>
<td>0.381</td>
<td>0.302</td>
<td>0.04</td>
</tr>
<tr>
<td>PLASTICO</td>
<td>0.1297</td>
<td>1.268</td>
<td>0.476</td>
<td>0.361</td>
<td>0.044</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.00</td>
<td>0.607</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.01</td>
<td>0.684</td>
<td>0.080</td>
<td>0.074</td>
<td>0.014</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.02</td>
<td>0.755</td>
<td>0.142</td>
<td>0.131</td>
<td>0.023</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.03</td>
<td>0.822</td>
<td>0.195</td>
<td>0.177</td>
<td>0.028</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.04</td>
<td>0.887</td>
<td>0.242</td>
<td>0.215</td>
<td>0.032</td>
</tr>
<tr>
<td>IMPOSTO</td>
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<td>0.949</td>
<td>0.284</td>
<td>0.246</td>
<td>0.035</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.06</td>
<td>1.005</td>
<td>0.323</td>
<td>0.271</td>
<td>0.037</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.07</td>
<td>1.057</td>
<td>0.359</td>
<td>0.291</td>
<td>0.038</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.08</td>
<td>1.106</td>
<td>0.393</td>
<td>0.309</td>
<td>0.040</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.09</td>
<td>1.153</td>
<td>0.425</td>
<td>0.323</td>
<td>0.041</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.10</td>
<td>1.199</td>
<td>0.455</td>
<td>0.335</td>
<td>0.042</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.11</td>
<td>1.236</td>
<td>0.476</td>
<td>0.345</td>
<td>0.043</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.12</td>
<td>1.253</td>
<td>0.476</td>
<td>0.354</td>
<td>0.044</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.1297</td>
<td>1.268</td>
<td>0.476</td>
<td>0.361</td>
<td>0.044</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.13</td>
<td>1.237</td>
<td>0.476</td>
<td>0.361</td>
<td>0.044</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.15</td>
<td>0.932</td>
<td>ROTTO</td>
<td>0.373</td>
<td>0.045</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.20</td>
<td>0.939</td>
<td>ROTTO</td>
<td>0.394</td>
<td>0.047</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.25</td>
<td>0.793</td>
<td>ROTTO</td>
<td>0.414</td>
<td>0.049</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.35</td>
<td>0.786</td>
<td>ROTTO</td>
<td>0.420</td>
<td>0.050</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.45</td>
<td>0.795</td>
<td>ROTTO</td>
<td>0.424</td>
<td>0.050</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.50</td>
<td>0.791</td>
<td>ROTTO</td>
<td>0.430</td>
<td>0.051</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.60</td>
<td>0.793</td>
<td>ROTTO</td>
<td>0.435</td>
<td>0.051</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.70</td>
<td>0.794</td>
<td>ROTTO</td>
<td>0.438</td>
<td>0.052</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>0.80</td>
<td>0.796</td>
<td>ROTTO</td>
<td>0.441</td>
<td>0.052</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>1.00</td>
<td>0.811</td>
<td>ROTTO</td>
<td>0.447</td>
<td>0.053</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>1.50</td>
<td>0.634</td>
<td>ROTTO</td>
<td>0.493</td>
<td>0.057</td>
</tr>
<tr>
<td>IMPOSTO</td>
<td>1.85</td>
<td>0.607</td>
<td>ROTTO</td>
<td>ROTTO</td>
<td>ROTTO</td>
</tr>
</tbody>
</table>

**Stress analysis of the reinforcements along the critical surface**

The following remarks can be highlighted:

- the rigid method gives higher values (SF=1.350) than the Displacements Method;
- with the Elastic limit method a much smaller value is found (SF=1.087);
- with the Plastic limit method, a value close to the rigid method is found (SF=1.268);
- Macstars W correctly runs the reinforcement stress analysis at increasing displacements, allowing to determine the SF-δ curve;
- For a displacement equal to 0 the SF value is the one without reinforcements (SF=0.607);
- for very large displacements (>1.85), at which corresponds the breaking/pullout of ALL reinforcements, SF is the same of the structure without reinforcements (SF=0.607)

7 – CRITICAL REMARKS ABOUT THE DISPLACEMENTS METHOD

Even if the reinforcement simulation through the ‘Displacement method’ represents for the reinforced slopes stability analysis an enormous improvement of the design approach, there are some remaining limits which is better to put in evidence:

A. The application of the ‘Displacement method’ requires circular surfaces for the stability analysis. Therefore it is not possible to apply it for situations where particular geotechnical and interaction conditions between reinforced geosynthetics and the soil, could create potential irregular settlement surface.

B. The safety factor related to the slope stability comes from equations valid for rigid equilibrium, that is without taking into consideration the deformative characteristics of the soils. This hypothesis could be too simple in presence of articulate stratifications

C. The compatibility of the reinforcement deformations with the soils one is not take into consideration: whatever could be the assigned displacement (or reinforcement deformation) the contribution to the slope stability offered by the soil is always the same.
TECHNICAL NOTE 2: GEOTECHNICAL PARAMETERS

DRAINED AND UNDRAINED CONDITIONS
In saturated soils volume changes can only occur if water drains from the pores. The speed at which water drains depends on the pore pressure gradients and especially on the permeability of the soil, which in turn depends principally on the grain sizes of the smallest grains.

- If soils drain quickly, drainage will occur during construction and the pore pressures are always in equilibrium with the groundwater and drainage conditions - DRAINED
- If soils drain slowly, there may not be time during construction for drainage to occur. In this case the volume and water content of the soil does not change - UNDRAINED

The effective stress principle is fundamental to design. All soil behaviour is governed by the effective stress \( \sigma' \) given by:

\[
\sigma' = \sigma - u
\]

where \( \sigma \) is the total stress and \( u \) is pore pressure. The total stress \( \sigma \) arises from the bulk unit weight of the soil and from any free water (stress applied to cracks).

The strength of a soil depends on its resistance to shearing stresses. Resistance to shear is provided by the friction between the soil particles and is proportional to the normal force between them:

\[
\tau_f = \sigma' \tan \phi' \quad \text{(Mohr – Coulomb failure criterion)}
\]

where

- \( \tau_f \) = shearing stress at failure
- \( \sigma' \) = effective normal stress
- \( \phi' \) = effective angle of friction (angle of shearing resistance)

Under certain conditions, clay soils exhibit a cohesive resistance to shearing \( c \). This is known in terms of effective stress as the apparent cohesion \( c' \):

\[
\tau_f = c' + \sigma' \tan \phi'
\]

Under undrained conditions, a saturated clay soil appears to have a constant limiting shear strength as the undrained cohesion or undrained shear strength \( c_u \) and

\[
\tau_f = C_u
\]

At failure, at a particular water content \( w_f \) the effective normal stress is \( \sigma'_f \) and the shearing resistance \( \tau_f \) is the strength. The soil will have different strength at different water contents and different normal effective stresses.

The principal differences between drained and undrained conditions and the implications for design calculations are:

- **Drained conditions**
  - Water drains during construction so water contents change. The pore pressures are in equilibrium and can be determined. Calculations are done in terms of effective stresses. The calculations are called drained or effective stress analyses.

- **Undrained conditions**
  - No water drains during construction and so water contents remain unchanged. Pore pressure are not in equilibrium and cannot be calculated easily. Calculations are done in terms of total stress. The calculations are called undrained or total stress analyses.

Soil strength
In general terms soil strength is the maximum shear which the soil can safely sustain. This depends on the effective stress, on the water content and on the displacement or strain which is allowed.

The three strength terms that we should be familiar with are:

- **PEAK STRENGTH**
  - This is the maximum shear stress. It occurs at relatively small strains (often about 1%) and therefore at relatively small displacements (often about 1mm). At the peak state the pore pressures and volumes are changing.

- **CONSTANT VOLUME STRENGTH (also called critical state, fully softened or ultimate strength)**
  - This is a strength when the soil continues to strain at constant shear stress and constant volume and pore pressure. The constant volume strength is reached after moderate strains (approx 10%) and after moderate displacement (approx 10 mm)

- **RESIDUAL STRENGTH**
  - This is the very smallest strength a soil can have. In clays in which the grains are flat and platy the residual strength may be as little as half the constant volume strength. In other cases in which the soil grains are more rounded or angular the residual strength is the same as the constant volume strength. In clays the residual strength is only reached after relatively large displacement.

**CHOICE OF STRENGTH FOR DESIGN**
We need to establish if the calculations are to be done for undrained conditions in terms of total stress using the undrained strength or done for drained conditions in terms of effective stress using the effective stress parameters. We also need to determine which strength, peak, constant volume or residual is appropriate for the design.

The selection of design parameters will depend on circumstances such as:
- the drained conditions in the soil;
- temporary or permanent structure;
- factor of safety, risks and consequences of malfunction.

Normally designs should be for drained conditions taking the worst set of pore pressure conditions envisaged. These will give safer designs than undrained analyses in most cases. If undrained analyses are only used the designer must be satisfied that there will not be any significant drainage during the design life of the wall.
If undrained analysis is used for walls or slopes, consideration should be given to the development of vertical cracks which may fill with water during rain. The design should be checked for the case where there is a full hydrostatic water pressure distribution on the active side unless it can be shown that this is impossible.

**PROJECT DATA: AVAILABLE TABLES**

In case the geotechnical data are uncompleted, it is possible to make reference to what is said in technical literature.

<table>
<thead>
<tr>
<th>Table 1. Unit weights of soils (and similar materials)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Gravel</td>
</tr>
<tr>
<td>Well graded sand</td>
</tr>
<tr>
<td>and gravel</td>
</tr>
<tr>
<td>Coarse or medium</td>
</tr>
<tr>
<td>sand</td>
</tr>
<tr>
<td>Well graded sand</td>
</tr>
<tr>
<td>Fine or silty sand</td>
</tr>
<tr>
<td>Rock fill</td>
</tr>
<tr>
<td>Brick hardcore</td>
</tr>
<tr>
<td>Slag fill</td>
</tr>
<tr>
<td>Ash fill</td>
</tr>
</tbody>
</table>

$\gamma_{\text{sat}} = \gamma_{\text{dry}} + n\gamma_{\text{w}}$

where $n$ is the soil porosity in %

The moist bulk weight $\gamma_{\text{dry}}$ and the saturated bulk one $\gamma_{\text{sat}}$ are correlated to:

The scale for strength estimated in the field is given in the following table. A scale in terms of undrained shear strength is as follows (from BS 5930)

<table>
<thead>
<tr>
<th>CLAY TERMS</th>
<th>$c_u$ (kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft</td>
<td>$&lt; 20$</td>
</tr>
<tr>
<td>Soft</td>
<td>$20 - 40$</td>
</tr>
<tr>
<td>Firm</td>
<td>$40 - 75$</td>
</tr>
<tr>
<td>stiff</td>
<td>$75 - 150$</td>
</tr>
<tr>
<td>Very stiff or hard</td>
<td>$&gt; 150$</td>
</tr>
</tbody>
</table>

Tab. 2: A scale of strength for clays

The relative density of sands and gravels may be determined by the standard penetration test.

<table>
<thead>
<tr>
<th>SAND AND GRAVEL TERMS</th>
<th>SPT (blows/ 300 mm penetration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose</td>
<td>0 – 4</td>
</tr>
<tr>
<td>Loose</td>
<td>4 – 10</td>
</tr>
<tr>
<td>Medium dense</td>
<td>10 – 30</td>
</tr>
<tr>
<td>Dense</td>
<td>30 – 50</td>
</tr>
<tr>
<td>Very dense</td>
<td>$&gt; 50$</td>
</tr>
</tbody>
</table>

Tab. 3 A scale in terms of N-values for sand and gravel (from BS 1377)

<table>
<thead>
<tr>
<th>Plasticity index (%)</th>
<th>$\phi'$ cr't (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>15</td>
</tr>
</tbody>
</table>

Tab. 4 Friction angle for siliceous sand and gravel and plasticity index (BS 8002)

The strength and stiffness of cohesionless soils are determined indirectly by in situ static or dynamic penetration tests. Details of three types of penetration tests as well as plate loading tests are given in BS 1377: Part 9. The peak and critical state angles of shearing resistance for siliceous sands and gravels may be estimated from the following equations:
The estimated peak effective angle of shearing resistance is given by:

$$\phi'_{\text{max}} = 30 + A + B + C$$

The estimated critical state angle of shearing resistance is given by:

$$\phi'_{\text{crit}} = 30 + A + B$$

The values of:
- A = angularity of the particles
- B = grading of sand/gravel
- C = results of standard penetration tests

are given in the following table (from BS 8002: 1994):

### Table 4: Friction angle of siliceous sands and gravels (from BS 8002: 1994)

<table>
<thead>
<tr>
<th>A - Angularity¹</th>
<th>A (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounded</td>
<td>0</td>
</tr>
<tr>
<td>Sub-angular</td>
<td>2</td>
</tr>
<tr>
<td>Angular</td>
<td>4</td>
</tr>
<tr>
<td>B - Grading of soil²</td>
<td>B (degrees)</td>
</tr>
<tr>
<td>Uniform</td>
<td>0</td>
</tr>
<tr>
<td>Moderate grading</td>
<td>2</td>
</tr>
<tr>
<td>Well graded</td>
<td>4</td>
</tr>
<tr>
<td>C - N³ (blows 300 mm)</td>
<td>C (degrees)</td>
</tr>
<tr>
<td>&lt; 10</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>60</td>
<td>9</td>
</tr>
</tbody>
</table>

¹ Angularity is estimated from visual description of soil.
² Grading can be determined from grading curve by use of:
   Uniformity coefficient = $D_{10} / D_{60}$
   where $D_{10}$ and $D_{60}$ are particle sizes such that in the sample, 10% of the material is finer than $D_{10}$ and 60% is finer than $D_{60}$.

### Table 5: Friction angle for rock (from BS 8002: 1994)

<table>
<thead>
<tr>
<th>Stratum</th>
<th>$\phi'$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chalk</td>
<td>35</td>
</tr>
<tr>
<td>Clayey marl</td>
<td>28</td>
</tr>
<tr>
<td>Sandy marl</td>
<td>33</td>
</tr>
<tr>
<td>Weak sandstone</td>
<td>42</td>
</tr>
<tr>
<td>Weak siltstone</td>
<td>35</td>
</tr>
<tr>
<td>Weak mudstone</td>
<td>28</td>
</tr>
</tbody>
</table>

NOTE 1. The presence of a preferred orientation of joints, bedding or cleavage in a direction near that of a possible failure plane may require a reduction in the above values, especially if the discontinuities are filled with weaker materials.

NOTE 2. Chalk is defined here as unweathered medium to hard, rabbly to blocky chalk, grade III (see Clayton, 1990).
1. Introduction
For the stability analysis of the slopes different calculation method exist, each one of them gives a final equation able to define the safety factor. Each method assumes a series of semplificatory hypothesis so to make the equation system solvable.

Some of these methods have been solved with iterative procedure, by creating a procedure which facilitates their implementation. Within these methods we are interested in examining the methods implemented by MACSTARS W, which are Bishop method (1955) and Janbu method (1954) or their simplification.

The stability analysis of these methods is the global limit equilibrum one. This verification is made by analysing a certain number of possible sliding surfaces in order to find the one that represents the minimum relation between available resistance at break and the effective one; the value of this relation constitutes the safety factor of the slope. So, chosen a failure surface the unstable part must be divided into slices and it must be studied the equilibrium of the single slice and then the global stability.

Given the high number of uncertainties, each method has semplificative hypothesis which make solvable the system, and these hypothesis make the difference between the methods. Here follows schematically the action of a single slice

To solve the system three static equation are available for each slice (vertical and horizontal displacement equilibrium) so for n slices you will have 3n linearly independent equations.

2. Bishop method
This method will adopt as first hypothesis the circular failure surface; moreover, it considers the results of the forces to be perpendicular to the slice lateral surface (\(X_i + X_{i+1} = 0\)). Using that hypothesis, it is possible to obtain a number of unknown equal to the equation one (3n equations in 3n unknowns).

By solving the system, you will obtain a safety factor given by the relation between the results of the stabilising moments and the results of the unstabilising ones, in this form:

\[
FS = \frac{\Sigma M_{stab}}{\Sigma M_{unstab}}
\]

3. Simplified Bishop method (used by Macstars W)
In this method one more hypothesis is added to the previous ones, that is the forces acting parallel to lateral surface of the slice are equal to 0. So, the system will be of 2n equations in 2n unknowns. The considered equations are the ones of the vertical movement equilibrium and of the moments, therefore the horizontal movement equilibrium is not granted.

The safety factor is:

\[
FS = \frac{\Sigma M_{stab}}{\Sigma M_{unstab}}
\]

4. Janbu method
The Janbu method is more general than the previous one, because it is applicable to non-circular failure surfaces (log-spirals, polygons, etc…).

The simplifying hypothesis, which excludes various uncertainties, is to fix the thrust surface along which slices interaction forces will act. This means that these forces are unknown, but given their application points, directions and lever arms are known. In this way are obtained 3n unknowns. By solving the system the safety factor is given, calculated as the ratio between the stabilising and unstabilising forces, in this form:

\[
FS = \frac{\Sigma F_{stab}}{\Sigma F_{unstab}}
\]

5. Simplified Janbu method (used by Macstars W)
In this case the above mentioned exemplifications are valid, moreover, the equilibrium of the acting forces on the slices lateral surface is supposed (\(X_i + X_{i+1} = 0\); \(E_i + E_{i+1} = 0\)).

A safety factor is always given in this form:

\[
FS = \frac{\Sigma F_{stab}}{\Sigma F_{unstab}}
\]

In this case, the whole moments equilibrium is not granted.
The simplified Janbu’s method leads to errors on the safe side in the calculated factor of safety FS up to 15%. The errors increase according to the ratio of depth to length of the slipped mass; for shallow slips the error is small.

Janbu recommends that the calculated factor of safety FS be multiplied by a correction factor $f_0$ which is related to the depth/length ratio of the slip surface as shown in the following figure:

$$ F = f_0 \times FS $$

**WARNING!**
The safety factor calculated by Macstars W is FS, therefore it is up to the user to manually calculate $F$.

6. Conclusions

It has been seen that the rigid Bishop and Janbu methods respect the three static equations, therefore their results are more than reliable. Moreover:

- The Bishop method is applicable only for circular failure surfaces and it seems to be a little more precise than the Janbu one; but the latter can be used also for not circular surfaces and so it can simulate a biggest quantity of failure surfaces.
- By analysing what has been said for the simplified methods, and stating the difference for the adopted sliding surfaces, in the simplified Bishop method, the global moments equilibrium is granted, but not that of the horizontal movement, the contrary is said for the Janbu one. Also in this case the Janbu method seems to be less precise, but more useful because it studies different kind of failure surface.

Here follows a table, which compares the safety factors of different methods (Fredlund and Krahn, 1977); the comparison could be made assuming as reference result the one of Mongenstern and Price method (1965), which results to be the more detailed method (even if of difficult mathematic solution).

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Example problem*</th>
<th>Ordinary method</th>
<th>Simplified Bishop method</th>
<th>Spencer’s method</th>
<th>Janbu’s simplified method</th>
<th>Janbu’s rigorous method**</th>
<th>Mongenstern-Price method $(\phi) = constant$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple 7:-1 slope, 8 ft (2.4 m) high, $\phi = 30^\circ$, $c = 600$ psi (4.1 kPa)</td>
<td>1.978</td>
<td>2.080</td>
<td>2.073</td>
<td>14.81</td>
<td>0.237</td>
<td>2.041</td>
</tr>
<tr>
<td>2</td>
<td>Same as 1 except with $\phi = 10^\circ$, $c = 0$</td>
<td>1.288</td>
<td>1.377</td>
<td>1.373</td>
<td>10.49</td>
<td>0.185</td>
<td>1.648</td>
</tr>
<tr>
<td>3</td>
<td>Same as 1 except with $c_r = 0.25$</td>
<td>1.607</td>
<td>1.766</td>
<td>1.761</td>
<td>14.33</td>
<td>0.255</td>
<td>1.735</td>
</tr>
<tr>
<td>4</td>
<td>Same as 1 except with $c_r = 0.25$ for both materials</td>
<td>1.029</td>
<td>1.124</td>
<td>1.118</td>
<td>7.93</td>
<td>0.139</td>
<td>1.191</td>
</tr>
<tr>
<td>5</td>
<td>Same as 1 except with a piezometric line</td>
<td>1.693</td>
<td>1.834</td>
<td>1.838</td>
<td>13.87</td>
<td>0.247</td>
<td>1.827</td>
</tr>
<tr>
<td>6</td>
<td>Same as 2 except with a piezometric line for both materials</td>
<td>1.171</td>
<td>1.248</td>
<td>1.245</td>
<td>6.88</td>
<td>0.121</td>
<td>1.333</td>
</tr>
</tbody>
</table>

*Width of slope is 0.05 (0.1 m) and the tolerance on the nonlinear solutions is 0.001.

**The line of thrust is assumed at 0.23.

We notice that these values are referred to the slope stability (without reinforcements) and therefore they could be different in case of reinforced soils structures.
These notes aim to remark some Macstars W method particularities adopted for the research of minimum Safety factor of a slope stability analysis.

The following considerations refer to the general option, that is the Global Stability, but the basic concept (i.e. the imprecision relative to the calculated safety factor) remains valid for any analysis made.

We have to remind that among the parameters required by Macstars W to run a Global Stability analysis there are:

- Range of Surfaces Initiation Points;
- Range of Surface Termination Points
- Total number of surfaces to analyse
- Total number of initiation point for the surfaced to be analysed.

Figure 1 shows the meaning of the above-referred parameters:

Actually the program considers a finite number of termination points; such a number is automatically calculated rounding up the number obtained dividing the quantity of surfaces by the number of initiation point.

Once the stability analysis is over, the software gives the minimum FS value among all the analysed surfaces.

If only one input parameter of the Stability Analysis surfaces is modified, different FS values will be obtained. This fact may generate doubts about the solver quality, but this is an unavoidable circumstance related to the problem we are dealing with, which cannot be related with the solver quality. We have to remind, in fact, that many engineering problems are solved trough numerical methods, whose results depend on the modelling operation (input parameters).

The control of the results is up to the engineer and, if necessary, he has to change the modelling (i.e. input parameters) in order to find the most severe solution.

Following, some considerations for a better understanding of the analytical aspects that cause the possible variations on the result and some practical advice for the modelling.

From the mathematical point of view, the FS safety factor for a slope stability is one of the most variable functions, depending on: geotechnical characteristics of soils, loads, slope geometry, etc. The problem is that for this function the definition law or derivative law are not known, therefore it is not possible to calculate the function FS minimum through the mathematical method given by the analysis: solution of the homogeneous system of the partial derivatives and analysis of the hessian’s 2nd order derivatives.

Once an u-uple combination for the variable FS function is assigned, it is not possible to determinate its value in a direct way: in order to determine FS, some preliminary calculations through a complicate algorithm are required, thus obtaining a solving equation whose result is FS.

As we have to accept the impossibility of knowing the actual minimum of such a function, it is still possible an approximate estimation of the minimum. A possible procedure is the following: once defined a research interval and a number of initiation/termination points to be analysed, one can calculate the FS function value for these points, assuming that the minimum of the function corresponds to the minimum of the chosen domain. We have to remark that a minimum calculated this way is necessarily approximate and the approximation margin depends on the regularity characteristics of the curve and on the effective number of the considered points.

As a further explanation of this subject we analyse some explicative graphs, developed by referring (on behave of simplicity) to one-variable function.

Figure 2 shows a generic curve on an interval. Assuming to calculate the minimum of the function with the above explained method, it will be possible to know the function value only in a finite number of points (only seven points on the case under examination). The
found minimum (5.165) is only an estimation of the effective minimum (4.999). The same considerations could be also done for the maximum of the function.

Figure 3 shows the same function analysed with a bigger quantity of points, thus obtaining a different minimum estimation, which happens to be more precise in this particular case.

The precision on result does not depend only on the number of points used to analyse the curve. The case in figure 4 presents the same case as figure 2, but with a different placement of the analysis points in the function domain. As we can see solution is now completely different.

The approximation margin of a minimum calculated by using this procedure depends, among other facts, on the curve regularity characteristics, as it is heavily affected by possible curve discontinuities. The following examples are similar to the ones already analysed in fig. 2-4, but in this case the function is discontinuous.
The same considerations made for figures 2 to 4 may be applied. The only difference for the cases shown in fig. 5 to 7 is the bigger mistake that can be obtained in the minimum’s estimation. This circumstance has a logic explanation: while a polygonal passing through assigned points of a continuous function can sufficiently approximate the real function, the polygonal passing through points of a discontinuous function is not fitted to approximate the function. Therefore, if the minimum is close to a discontinuity point, the result obtained by searching the minimum by means of a discrete scanning of the function is strongly affected by the choice of the scanning parameters.

Going back to the Stability Analysis problem, the multi-variable FS function may be, depending on the situation, very irregular with discontinuities and steep gradients. In particular the FS function is very sensitive to the geometrical parameters of the slope and of the possible sliding surface and this sensitivity is stronger for almost cohesionless soils; furthermore, the presence of reinforcements emphasises the function's oscillations and gradient. If for the reinforcements a minimum anchorage length is required (i.e. Macstars W), the FS function becomes discontinuous: in fact such a circumstance means, from an analytical viewpoint, that there is an instantaneous strength loss.

From a practical point of view there is not an absolute procedure to follow in order to avoid the problem:

an engineering approach has to be used, by giving reasonable input parameters targeted on the expected critical zone or, if it is non possible to individuate the critical zone, by running some preliminary checks in order to find the critical areas to be targeted by further checks.
INTRODUCTION
This technical note is targeted at providing advise on the parameter Ru, the constant pore water pressure (pwp) ratio parameter and its influence on the stability analysis.

In a constant, homogeneous, regular slope the pore water pressure can be specified by identifying a piezometric surface. Ru is the ratio of the pore pressure at a point to the overburden pressure at that point. There is a clear linear relationship between the Safety Factor (SF) of a slope and the value of pwp ratio within a regular homogeneous slope.

The pressure of the pore water is measured relative to the atmosphere and the level at which the pressure is atmospheric (i.e. zero) is defined as the Water Table (WT) or the phreatic surface.

Below the water table the soil is assumed to be fully saturated although it is likely that, due to the presence of small volumes of entrapped air, the degree of saturation will be marginally below 100%.

WT levels may change according to climatic conditions or construction operations. Perched water tables can occur locally, contained by soils of low permeability. Artesian conditions may exist where the pressure in the artesian layer is governed not by the local water table but by a higher water table level at a distant location.

Below the water table the pore water may be static, the pressure depending on the depth below the water table, or may be seeping through the soil under hydraulic gradient; the later will be covered under WI-9.

From TN-2 we have the equation:

\[ \sigma' = \sigma - u \]  

Where:
\[ \sigma = \text{total normal stress} \]
\[ u = \text{pore water pressure} \]
\[ \sigma' = \text{effective normal stress} \]

This principle applies only to fully saturated soils.

Consider a soil mass having an horizontal surface with the water table at surface level. The total vertical stress (the total normal stress on an horizontal plane) at depth \( z \) is equal to the weight of all materials (solids+water) per unit area above the depth, i.e.

\[ \sigma_v = \gamma_{sat} z \]  

the pore water pressure at any depth will be hydrostatic since the void space between the soil particles is continuous, therefore at depth \( z \):

\[ u = \gamma_w z \]  

rearranging equation (1) we get

\[ \sigma'_v = \sigma_v - u \]  

and substituting \( u \) and \( \sigma_v \) you obtain

\[ \sigma'_v = (\gamma_{sat} - \gamma_w) z = \gamma' z \]  

where \( \gamma' \) is the buoyant unit weight of the soil.

STABILITY ANALYSIS
The break of a slope of limited high and made for major part of cohesive soils, occur generally along surfaces with variable curving ray, with a maximum value in the middle a minimum value in the top and an intermediate value in the bottom; these values are similar to elliptical arches.

In the stability checks, usually is introduced the simplification that the breakage surface could be made similar to a circular arch.

A test method sufficiently accurate for a lot of situation is the Bishop one (see TN-3) in which the safety factor \( F \) could be determined through this relation:
Equation (6) could be conveniently re-written in another form through the proportionality coefficient of the neutral pressure $r_u$

$$r_u = \frac{u}{\gamma' h} \quad \text{that value seems to be constant along the whole sliding circle.}$$

When considering a slice in a slope stability situation:

$$\gamma' h = \frac{W}{b} \quad r_u = \frac{u}{(W/b)}$$

the expression $(W-u_b) tan \phi'$ of equation (6) become

$$W (1-r_u) tan \phi' \quad (8)$$

Obtaining the definitive equation

$$F = \frac{1}{\Sigma W \sin \alpha} \sum \left[ c' b + (W - u b) tan \phi' \frac{sec \alpha}{1 + \frac{tan \alpha tan \phi'}{F}} \right] \quad (9)$$

As $W (1-R_u) tan \phi$ is part of the resisting moment function, the larger the better. As $R_u$ increases this part of the function decreases and the FoS reduces.

$R_u$ values have been used for many years as a rapid mans of assessing the stability of a slope. However the means of estimating an average value of $R_u$ for a slope can be laborious and inaccurate. Representing the pore pressure conditions beneath the whole of a slope by a single value can be inappropriate. This method of approach may be frowned on by certain higher level analytical geotechnical engineers however it is currently seen as a practical way to progress our designs.

The general solution assumes that $R_u$ is constant throughout the cross-section and this is called the homogeneous pore pressure distribution.

$R_u$ VALUES TO BE ADOPTED FOR THE CALCULATION

Macstars W let the users to insert $R_u$ different from 0. In the first instance these values could be used:

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>$R_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular</td>
<td>0</td>
</tr>
<tr>
<td>Dry Cohesive</td>
<td>0.1</td>
</tr>
<tr>
<td>Wet Cohesive</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- Generally $R_u = 0$ should be used
INTRODUCTION
This note explains the factors implemented in the database of Macstars W in the Reinforcement Menu for the general data of the reinforcement and in the Standard menu - multipliers, for data related to a specific standard.

- Friction factor reinforcement-reinforcement, reinforcement-gravel, reinforcement-sand, reinforcement-silt and reinforcement-clay;
- Safety factor for friction of filling material made of gravel, sand, silt and clay;
- Safety factor for breakage for filling material made of gravel, sand, silt and clay.

A - PULLOUT COEFFICIENTS
The pullout coefficients derive both from experimental results directly made by OM (so it is for Terramesh, whose values have been obtained through pullout tests) and from information given by the manufacturers (i.e. Linear Composites). For the missing data we have made reference to extrapolations.

The pullout coefficient between reinforcement and structural embankment material, derives from the following considerations:

The necessary force to get the pullout of the reinforcement unthreading \( F_{po} \) is given by the relation:

\[
F_{po} = 2 \sigma v L W \mu \tan \Phi
\]

- \( \Phi \) = Friction angle of the structural embankment material
- \( \mu \) = Interaction factor between structural embankment material and reinforcement
- \( L \) = Embedded length of the reinforcement
- \( W \) = Reinforcement’s width
- \( \sigma v \) = Vertical pressure

The \( F_{po} \) force is experimentally determined for each kind of filling material (gravel, sand, silt and clay); from the knowledge of the other values it is possible to find the interaction factor \( \mu \) which is normally provided by every reinforcement’s manufacturer.

For the pullout factors of the Terramesh here are the \( \mu \) values:

<table>
<thead>
<tr>
<th>Terramesh</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
<th>Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.3</td>
<td>0.5</td>
<td>0.65</td>
<td>0.9</td>
</tr>
</tbody>
</table>

B - PULLOUT SAFETY FACTOR
For the “NO STANDARD” section, the pullout safety factor is equal to 1 for each product.

C - NOMINAL STRENGTH AND BREAKING SAFETY FACTOR
The values given to the reinforcement, for ‘no standard’ conditions, have been assumed equal to the ones calculated following the BS8006 for 60 years, choosing the most severe situation between “Retaining Walls” and “Reinforced Slopes”:

**Terramesh 8x10/2.7 mm**

<table>
<thead>
<tr>
<th>SAFETY FACTOR:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_m ) = 1.30 (for clay, silt and sand)</td>
</tr>
<tr>
<td>( f_m ) = 1.44 (for gravel)</td>
</tr>
</tbody>
</table>

Now it should be sufficiently clear the meaning of the report’s part related to the characteristics of the reinforcement:
TECHNICAL NOTE 7: PARTIAL FACTORS FOR TERRAMESH 8X10/2.7 PVC

INTRODUCTION

The procedure for determining suitable reinforcement properties is described in Annex A of BS 8006. Partial factors are ascribed to each of the potential strength reducing influences to produce a total materials factor \( f_m \). This is applied to the reinforcement "base strength", \( T_B \), such that

\[
T_D = \frac{T_B}{f_m}
\]

Where \( T_D \) is the tensile strength of the reinforcement to be used in the design.

1- Reinforcement base strength, \( T_B \)

The value of \( T_B \), the reinforcement base strength, for metallic reinforcement should be the ultimate tensile strength based on net cross-sectional area. On the basis of the tensile strength tests carried out at CTC, Denver-USA in accordance to ASTM A-975, the following average value has been found:

\[
T_B = 50.11 \text{ kN/m}
\]

It has to be noted that this value is the result of tensile strength tests on mesh panels with prevented lateral contraction and this explains why the value is bigger than 47 kN/m

2- Partial materials factor, \( f_m \)

The partial materials factor, \( f_m \), is made up of a number of sub-factors such that

\[
f_m = f_{m1} \times f_{m2} \times f_{m21} \times f_{m22}
\]

- \( f_{m1} \) is a factor relating to the manufacturing process
- \( f_{m2} \) is a factor relating to the extrapolation of data
- \( f_{m21} \) is a factor relating to the damage caused to the products during the installation process
- \( f_{m22} \) is a factor relating to the effects of the environment on the products.

ASSESSMENT OF PARTIAL FACTORS FOR THE TERRAMESH

The following considerations relate to paragraph 5.3.3 (Reinforcement materials partial factors) and the Appendix A from the code BS 8006.

Two basic materials partial factors \( f_{m1} \) and \( f_{m2} \) are applicable to the reinforcements. The factor \( f_{m1} \) is related to the properties of the materials itself whereas \( f_{m2} \) is concerned with construction effects and environmental effects.

The total material factor \( f_m \) is given by:

\[
f_m = f_{m1} \times f_{m2}
\]

where

\[
f_{m1} = f_{m11} \times f_{m12}
\]

\[
f_{m2} = f_{m21} \times f_{m22}
\]

\( f_{m1} \) - Product Manufacture

This factor is a combination of:
- whether or not a standard for specification, manufacture and control testing of the base material exists (\( f_{m111} \))
- whether or not standards exist for the dimensions and tolerances of the particular product being manufactured (\( f_{m12} \))

\( f_{m11} \)

By following the approach required for polymeric reinforcements (in order to take into account the real test results distribution), reference is made to the mean base strength (the characteristic being the 95th percentile value) in the case of the mean base strength, \( f_{m11} \), is determined as follows:

\[
f_{m11} = 1 + \frac{1.64 \sigma}{\mu - 1.64 \sigma}
\]

where

\[
\mu = \text{the mean reinforcement base strength} = 50.11 \text{ kN/m}
\]

\[
\sigma = \text{the standard deviation of the reinforcement base strength} = 2.301
\]

therefore

\[
f_{m11} = 1.081
\]

\( f_{m12} \)

As the reinforcement base strength depends upon the tolerances on the cross section, which depend upon the wire diameter tolerances, a factor bigger than unity has to be used.

As the tolerances on the wire 2.7 mm diameter are ± 0.06 mm (as per EN 10223-3 Table 1), the corresponding ratio between nominal (5.72 mm² for 2.7 mm diameter) and minimum (5.47 mm² for 2.64 mm) area is 1.04 and we finally have

\[
f_{m12} = 1.04
\]

Thus having

\[
f_{m1} = f_{m11} \times f_{m12} = 1.081 \times 1.04 = 1.124
\]
**fm12 - Extrapolation**

Extrapolation covers the combination of:

- the assessment of available data in order to derive a statistical envelope \( (fm_{121}) \)
- the extrapolation of this statistical envelope over the required service life \( (fm_{122}) \)

\( fm_{121} \)  
Relates to the assessment of quality, quantity and duration of available data. \( fm_{121} \) represents a measure of the confidence in the available data for large amounts of directly relevant data available over a long period of time, the statistical analysis would permit a value of 1.0 for \( fm_{121} \). A value of 1.0 can be adopted for the Terramesh reinforcements, due to the extensive tests carried out over many years.

\( fm_{122} \)  
This involves the extrapolation of the available data over a longer period up to the required service life of the structure. A value of 1.05 can be adopted for the Terramesh reinforcements, due to the 100 years experience with wire mesh applications.

\[ fm_{12} = fm_{121} \times fm_{122} = 1.05 \]

**fm21 – Installation**

The partial safety factors for mechanical damage during installation are given in the table below, based on the type of fill used. These assume that the fill is well graded and uniform. Some fills, e.g. crushed rocks, may be require that a protective blinding of granular material be placed prior to and following placing of the PVC-U mesh to avoid damage to the coating. Galvanised steel is not normally damaged during the construction process.

The component factors for damage cover a combination of:

- the short term effects of damage prior to and during installation, \( fm_{211} \)
- the long term effects of the damage, \( fm_{212} \)

\( fm_{211} \)  
Galvanized steel is not usually damaged during the construction process in a short term effect if the material comply with the common standards about the fill materials accepted for reinforced soil structures.

The protection offered by the galvanising is a chemical process, affecting the parent metal itself and is not a coating such as epoxy coating or painting. Galvanising is a self healing process with any indentations creating an electrochemical process of self repair. The PVC coating is extruded on the steel wire used in the Terramesh which is made of galvanised wire heavily zinc coated and therefore it is in accordance with the above considerations. A value of 1.0 can be adopted for the Terramesh reinforcements.

\[ fm_{211} = 1.0 \]

\( fm_{212} \)  
A maximum value of 1.165 can be adopted for the Terramesh, assuming to use for the structural fill the worst soil grading (0-50 mm) as reported on the above mentioned enclosure. Soil gradings beyond these values (up to the 0-200 mm) are acceptable provided that a protective blinding of granular material be placed prior to and following placing of the PVC-U mesh to avoid damage to the coating. A summary of the test results is reported in the following table

<table>
<thead>
<tr>
<th>Fill material</th>
<th>Max particle size (mm)</th>
<th># damages (per m²)</th>
<th>( fn_{212} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silts and clays</td>
<td>0.06</td>
<td>0</td>
<td>1.05</td>
</tr>
<tr>
<td>Sands</td>
<td>7</td>
<td>0</td>
<td>1.05</td>
</tr>
<tr>
<td>Gravel</td>
<td>50</td>
<td>4</td>
<td>1.165</td>
</tr>
</tbody>
</table>

For soil gradings within the range reported, intermediate values between 1.05 (conservative assumption) and 1.165 of such coefficient can be adopted.

\[ fn_{212} = 1.05 \text{ - } 1.165 \]

\[ fm_{21} = fm_{211} \times fm_{212} = 1.0 \times (1.05 \text{ or } 1.165) = 1.05 \text{ or } 1.165 \]

**fm22 Environmental Component Factor**

In the case of environmental considerations, the component factor allows for a different rate of degradation compared with the anticipated losses from the statistical studies (i.e. from \( fm_{12} \)). This component factor allows for uncertainties in material behaviour whilst stressed in the soil during the service life of the structure. It reflects the sensitivity of the material to its environment whilst carrying the design loads. The PVC coating in the Terramesh reinforcements used to protect the steel wire is not subject to particular tensile stresses because incomparably more deformable than its steel wire core. The PVC material is proved to be chemically not aggredible when used in environments characterised by a pH greater than 2.5 under these conditions we can therefore assume a value of 1.05 for the Terramesh reinforcements.

\[ fm_{22} = 1.05 \]

**CONCLUSIONS**

The following table recaps the partial factors adopted by Macstars W

<table>
<thead>
<tr>
<th>( fm_{111} )</th>
<th>( fm_{112} )</th>
<th>( fm_{121} )</th>
<th>( fm_{122} )</th>
<th>( fm_{211} )</th>
<th>( fm_{212} )</th>
<th>Total ( fm )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean base strength</td>
<td>Tolerances on cross section area</td>
<td>ISO 9002 certified</td>
<td>100 years of available data</td>
<td>No short term effect of construction damage</td>
<td>Maximum value for granular fills</td>
<td>No degradation of the PVC coating</td>
</tr>
<tr>
<td>1.081</td>
<td>1.04</td>
<td>1.00</td>
<td>1.05</td>
<td>1.00</td>
<td>1.05</td>
<td>1.05</td>
</tr>
</tbody>
</table>

\( fm_{111} \) Quality control  
\( fm_{112} \) Tolerances in manufacturing  
\( fm_{121} \) Confidence in the available data  
\( fm_{122} \) Confidence in extrapolation to design life
Therefore the Long Term Design Strength (LTDS) calculated according to BS 8006 – Annexe A is equal to

\[
TD = T_b/f_m = \frac{50.11}{1.30} = 38.5 \text{ kN/m} \\
LTDS \text{ (in clay, silt, sand)}
\]

\[
TD = T_b/f_m = \frac{50.11}{1.44} = 34.8 \text{ kN/m} \\
LTDS \text{ (in gravel)}
\]
TECHNICAL NOTE 8: TIPS FOR THE STABILITY CHECKS OF RSS

INTRODUCTION
This note wants to give some practical tips about the way to run stability checks with Macstars W with respect to these aspects:

- How to choose kind and distribution of the reinforcements for the internal, external and global stability checks
- Research of critical surface: which kind of surface and how many trial surfaces
- Modelling of pore water pressures

HOW TO CHOOSE KIND AND DISTRIBUTION OF THE REINFORCEMENTS FOR INTERNAL STABILITY
The internal stability check is the first to be done taking into consideration the effective distribution of the tensions within a reinforced slope, of the construction method and of the kind of structure.

For retaining walls the full scale tests show that the axial tension of the reinforcement increase from top to bottom and decrease where the structure meets the foundation soil; moreover in a same reinforcement the axial tension changes, moving from the face to inside, with one or more maximum localised to different distances, in relation to the height of the reinforcement and the presence of surcharge loads. Generally in a steep face structure, the points of maximum axial tension in the reinforcement has a shape similar to a logarithmic spiral. And that for two reasons:

1. Increase of the horizontal pressures in the soil with the height
2. Layer by layer construction of the structure which creates a reinforcement deformation, which within the time, interests higher reinforcements so to mobilise strength only in relation to deformation produced by what has been build over them. So, if there are no surcharge load at the top of the structure, the horizontal deformations are null and tends to increase with the depth.

The limit equilibrium analysis, especially for rigid method is not able to simulate in a reliable way the structure tensional state. Such methods can only provide the required stabilising force, without giving information on their distribution within the reinforcement.

Variation of the maximum tensions

Distribution of the axial tension in the reinforcement and its zone of action
A correct approach for the design of the reinforced soil structures, according to the characteristics of Macstars W could be the following:

1-Blocks division
Once defined the height of the reinforced soil, make a first block division (the number is related to the height), giving to each block an anchoring length of 0.7 H.

2-Check and optimisation of each block
Each block is checked against internal and external stability

Going on with the test to the following block, it could be necessary to increase the reinforcement strength, to add some more reinforcement to increase the height of the block, to decrease the above one or increase the length:

Before going on, it is always better to verify stress rate and behaviour of the reinforcement through the results window where I can determine if they are working at pullout or breaking conditions (it is useless increase the reinforcement resistance if they are in pullout conditions)
I can increase the number of the reinforcement or increase their strength.

The choice among different possibilities will be done on the base of economical convenience. At the end of the optimization process, the structure will be as the one shown in the following picture: length and strength increasing with the depth; the lower reinforcement could be shorten and less resistant because less solicited than the others, unless there are global stability problems.

The same structure could be built in a different way, but does it make sense to have 7m long reinforcements in order to grant the stability of a failure mechanism passing at the bottom?
At the end of the internal stability checks, the external one should be made.

If the check is not satisfied, the designer will lengthen or change the base reinforcements.

Final global stability check
The proposed approach is surely useful to the retaining work, while for a reinforced slope with an angle smaller than 45-50 °, stress conditions in the reinforcements are more uniform, and it is generally valid to adopt an uniform length for the reinforcements.
RESEARCH OF THE CRITICAL SURFACE: INFLUENCE OF THE NUMBER OF SURFACES AND OF THEIR SHAPE

The safety factor can vary in relation to the parameters used for the research of the critical surface. It is important to find the right combination between kind of surface, dimension of the segment and number of surface. The parametrical study of the variation of Fs in relation to those values, shows us some typical aspects of the search of the critical surface:

- A too little length of the segment for polygonal surfaces, or too big for circular ones, gives Fs values not reliable;
- The circular surfaces are poorly affected by the number of tentative surfaces, the contrary is valid for the polygonal ones;

<table>
<thead>
<tr>
<th>Surface Type</th>
<th>Minimum FS</th>
<th>Maximum FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygonal</td>
<td>1.374; L=3.5; N=4000</td>
<td>2.382; L=0.5; N=100</td>
</tr>
<tr>
<td>Circular</td>
<td>1.40; L=0.5; N=4000</td>
<td>2.472; L=3; N=100</td>
</tr>
</tbody>
</table>

Generally, once found a more critical kind of surface, for the examined problem, analysing for example only 500 surfaces, and varying length, segment and shape, the number is increased and almost all the possible Fs value are found.

In effect the shape of the critical surface and the number of trial surfaces necessary to find the possible minimum Fs value are different for each case and could be strongly influenced by the geometry of the slope, the presence of a bedrock and the reinforcement pattern (the safety factor is a multivariable function characterised by an high discontinuity, see TN 4) therefore there is no rule and it is necessary to use experience and common sense in choosing the input data. It is not a good practice to make analysis with less than 1000 tentative surface, it must not be used research intervals too big with respect to the number of trial surface, it is necessary to significantly increase the number of surfaces for complex problem. This statement is confirmed, for a low number of trial surface, by heavy variations of the maximum safety factor of polygonal surfaces with the length of the segment and with the number of trial surfaces.
MODELLING OF THE PORE WATER PRESSURE REGIME

Macstars W allows to simulate all the regime conditions of the pore water pressure. In the ‘Piezometric Surface’ window we can input X and Y coordinates of the polygonal which approximate the free surface of the water table (both static and dynamic). For every point it is possible to insert the water table basis ordinate (for perched WT) if it is not done, the basis is put on the lower layer of the soil. If it is necessary to model a perched WT the user must define, apart from X,Y, of each point, also the ordinate of the water table basis. For artesian WT the 4th required value is the pressure in the water table.

Perched water table

Artesian water table
TECHNICAL NOTE 9: SETTLEMENTS CALCULATION

Once the variation profile of the induced stresses is known, the settlements calculation is obtained by applying the elasticity theory to single layers of granular soils, and the Terzaghi monodimensional consolidation theory for the cohesive soils.

The settlements calculation is extended in depth till the vertical tensional variation due to the load ($\Delta\sigma_z$) is lower than 10% of the initial geostatic tension.

In the calculation the cohesive and uncohesive soils are distinguished.

**GRANULAR SOILS: calculation following the elasticity theory, valid for small deformations**

The calculation of the settlement related to the $i$-th section ($s_i$) is given by the relation:

$$s_i = \left[ \Delta \sigma_z - \nu_i \left( \Delta \sigma_x + \Delta \sigma_y \right) \right] \cdot \Delta h_i / E_i$$

where:

$\Delta \sigma_z, \Delta \sigma_x, \Delta \sigma_y$ = variation of the vertical tensional state ($z$) and horizontal ones ($x, y$) in the $i$-th soil section

$\nu_i$ = Poisson coefficient in the $i$-th soil section

$E_i$ = deformability average modulus in the $i$-th soil section

$\Delta h_i$ = thickness of the $i$-th soil section

The elasticity $\nu$ and $E$ parameters are given directly by the user.

### Table 2-6 Typical range of values for the static stress-strain modulus $E_s$ for selected soils

<table>
<thead>
<tr>
<th>Soil</th>
<th>ksf</th>
<th>Mpa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very soft</td>
<td>50-250</td>
<td>2-15</td>
</tr>
<tr>
<td>Soft</td>
<td>100-500</td>
<td>5-25</td>
</tr>
<tr>
<td>Medium</td>
<td>300-1000</td>
<td>15-50</td>
</tr>
<tr>
<td>Hard</td>
<td>1000-2000</td>
<td>50-100</td>
</tr>
<tr>
<td>Sandy</td>
<td>500-5000</td>
<td>25-250</td>
</tr>
<tr>
<td>Glacial till</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>200-3200</td>
<td>10-15</td>
</tr>
<tr>
<td>Dense</td>
<td>3000-15000</td>
<td>144-720</td>
</tr>
<tr>
<td>Very dense</td>
<td>10000-30000</td>
<td>478-1440</td>
</tr>
<tr>
<td>Loess</td>
<td>300-1200</td>
<td>14-57</td>
</tr>
<tr>
<td>Sand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silty</td>
<td>150-450</td>
<td>7-21</td>
</tr>
<tr>
<td>Loose</td>
<td>200-500</td>
<td>10-24</td>
</tr>
<tr>
<td>Dense</td>
<td>1000-1700</td>
<td>48-81</td>
</tr>
<tr>
<td>Sand and gravel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>1000-3000</td>
<td>48-144</td>
</tr>
<tr>
<td>Dense</td>
<td>2000-4000</td>
<td>96-192</td>
</tr>
<tr>
<td>Shale</td>
<td>3000-300000</td>
<td>144-14400</td>
</tr>
<tr>
<td>Silt</td>
<td>40-400</td>
<td>2-30</td>
</tr>
</tbody>
</table>

### Table 2-7 Typical range of values for Poisson’s ratio $\mu$

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay, saturated</td>
<td>0.4–0.5</td>
</tr>
<tr>
<td>Clay, unsaturated</td>
<td>0.1–0.3</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>0.2–0.3</td>
</tr>
<tr>
<td>Silt</td>
<td>0.3–0.35</td>
</tr>
<tr>
<td>Sand (dense)</td>
<td>0.2–0.4</td>
</tr>
<tr>
<td>Coarse (void ratio = 0.4–0.7)</td>
<td>0.15</td>
</tr>
<tr>
<td>Fine-grained (void ratio = 0.4–0.7)</td>
<td>0.25</td>
</tr>
<tr>
<td>Rock</td>
<td>0.1–0.4 (depends somewhat on type of rock)</td>
</tr>
<tr>
<td>Loess</td>
<td>0.1–0.3</td>
</tr>
<tr>
<td>Ice</td>
<td>0.36</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Different values of the axial deformation modulus and of the Poisson coefficient for different kinds of soils. Noted that is 0.5 for saturated clays (from Bowles: Foundation Analysys and Design, 1982)

**CONSOLIDATION SETTLEMENTS**

This settlements take place in conditions of filtration flow in transitory regime after the total tensions increment in a saturated cohesive soil. The program lets to calculate the primary settlement, but not the secondary one (creep) nor the instantaneous one.

As the soils have memory of their tensional history, the same soil will act in a different way, depending on the level of maximum load to which it has been subjected in the past: preconsolidation effective vertical tension.

With reference to the following picture (index variation diagram of the load increase) a soil which didn’t experiment a consolidating tension higher than the actual effective vertical tension (normalconsolidated soil, NC) will settlement following the T1 section. A soil subjected to an higher tension (overconsolidated, OC), will firstly follow the T2 re-compression section, till reaching the strength of preconsolidation after that it will go on with the T1 section.
Therefore to determine the settlement value it is necessary to know the parameters that characterise the possible settlement:

- **CR** = first compression ratio
- **RR** = re-compression ratio
- **σ′<sub>c</sub>** = pre-consolidation strength.

These parameters are found in the edometric tests, plotting the settlement data for all the loading increases in a semi-logarithmic diagram. The following one is the reaction that links the parameters:

**NC Soils**

\[ s_{adI} = C_C \cdot \log_{10} \left( \frac{\sigma'_{s} / \sigma''_{s}}{\sigma'_{o}} \right) \cdot \Delta h_i \]

**OC Soils**

\[ \begin{align*}
\sigma'_{s} &> \sigma'_{c} \quad \text{and} \quad \sigma'_{f} > \sigma'_{c} \quad s_{adI} = [C_C \cdot \log_{10} (\sigma'_{s} / \sigma''_{s}) + C_C \cdot \log_{10} (\sigma'_{f} / \sigma'_{o})] \cdot \Delta h_i \\
\sigma'_{s} &< \sigma'_{c} \quad \text{and} \quad \sigma'_{f} < \sigma'_{c} \quad s_{adI} = C_C \cdot \log_{10} (\sigma'_{s} / \sigma''_{s}) \cdot \Delta h_i
\end{align*} \]

The above formulation (which derive directly from Terzaghi consolidation theory), are used by Macstars W.
with the correction proposed by Skempton and Bjerrum:

\[ s_{ed,i} = \text{edometric settlement of the } i\text{-th layer} \]
\[ \mu_i = A + \alpha (1-A) = \text{correction factor of the } i\text{-th layer} \]
\[ A = \text{Skempton's parameter} \]
\[ \alpha = \text{adimensional coefficient} \]

As per the values to be assumed for \( A \) ed \( \alpha \):

**VALORI TIPICI DI A PER S=100\% (BJERRUM, 1957)**

- Very soft clays  \> 1
- Normal consolidated clays  0.5 - 1
- Overconsolidated clays  0.25 – 0.5
- Strongly overconsolidated clays  0 – 0.25

The \( \alpha \) coefficients depends upon the geometry and the soil characteristics and can be obtained through a back-calculation (if the foundation characteristics, \( \mu \) and \( A \) are known) using the following diagram (Lancellotta: Geotecnica, 1987):
TECHNICAL NOTE 10: MACSTARS W VALIDATION

The program Macstars W has been subjected to different numerical comparisons in terms of manual calculations vs. slope stability analysis software, in order to check the validity of its results.

The researches have compared the results obtained by Macstars and those of:

- Manual calculation
- software PANGEO-Pendii
- software SLOPE-W
- software TALREN

1 – COMPARISON WITH MANUAL CALCULATION USING JANBU METHOD
The comparison has been realised using the Janbu method, by prefixing the sliding surface and studying a simple case with only 2 slices [1].

![Scheme used in the manual calculation](image)

Fig. 1: Scheme used in the manual calculation

The comparison results are very close (maximum difference equal to 1.4%), but for the case 5 (difference equal to 13%), in which have a big place the different hypothesis of load diffusion within the embankment between manual calculation (it transfers the load, applied in surface, at the base of the slices) and Macstars W (which use a semi-corner of approximately 27° from the direction of the load)

<table>
<thead>
<tr>
<th>CASE</th>
<th>CALCULATION SCHEME</th>
<th>FS Macstars</th>
<th>FS Manual</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Natural</td>
<td>1.341</td>
<td>1.340</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Uniform horizontal surcharge on the slope</td>
<td>3.917</td>
<td>3.918</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Uniform surcharge perpendicular to the slope</td>
<td>2.339</td>
<td>2.339</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Point loads perpendicular to the slope, at the slice centre</td>
<td>1.940</td>
<td>1.939</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Point loads at 25° to the slope, at the slice centre</td>
<td>1.150</td>
<td>0.994</td>
<td>+13.5</td>
</tr>
<tr>
<td>6</td>
<td>Point load perpendicular to the slope, at the 1st slice centre</td>
<td>1.640</td>
<td>1.639</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Horizontal water table</td>
<td>0.678</td>
<td>0.664</td>
<td>+2</td>
</tr>
<tr>
<td>8</td>
<td>Inclined water table</td>
<td>0.706</td>
<td>0.696</td>
<td>+1.4</td>
</tr>
<tr>
<td>9</td>
<td>With reinforcement</td>
<td>1.971</td>
<td>1.971</td>
<td>0</td>
</tr>
</tbody>
</table>

Tab. 1: comparison with manual calculation

2 – COMPARISON WITH THE CALCULATION PROGRAM PANGEO – PENDII
Pangeo-Pendii is a program which allows to calculate the safety factor along possible slope failure surface using the limit equilibrium method. [1]

The code considers the presence of anisotropic, homogenous and stratified soils, cohesive or non-cohesive, the presence of water table and eventual external loads as superficial seismic actions and tiebacks. Verifications have been carried out with different situation of horizontal and inclined water table and in different conditions of surcharge, using infinite homogenous slopes with an 25° inclination in gravel, sand, lime and clay, with or without reinforcement.
The verification has been done with the Bishop and Janbu methods using an assigned failure surface and the results have been reported in the following table in relation to the hypothesis with gravel and clay.

<table>
<thead>
<tr>
<th>Cases with water table (Bishop – Clay)</th>
<th>FS Macstars</th>
<th>FS Pendii</th>
<th>∆ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope totally submerged – still water table</td>
<td>2.237</td>
<td>2.226</td>
<td>+0.3</td>
</tr>
<tr>
<td>Slope partially submerged – still water table</td>
<td>1.703</td>
<td>1.678</td>
<td>+0.3</td>
</tr>
<tr>
<td>Slope out of layer – still water table</td>
<td>1.912</td>
<td>1.902</td>
<td>+0.5</td>
</tr>
<tr>
<td>Slope totally in filtering</td>
<td>0.69</td>
<td>1.026</td>
<td>-48.7</td>
</tr>
<tr>
<td>Slope partially in filtering</td>
<td>0.977</td>
<td>1.145</td>
<td>-17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cases with water table (Bishop – Gravel)</th>
<th>FS Macstars</th>
<th>FS Pendii</th>
<th>∆ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope totally submerged – still water table</td>
<td>2.942</td>
<td>2.93</td>
<td>+0.4</td>
</tr>
<tr>
<td>Slope partially submerged – still water table</td>
<td>2.306</td>
<td>2.286</td>
<td>+0.9</td>
</tr>
<tr>
<td>Slope out of layer – still water table</td>
<td>2.549</td>
<td>2.542</td>
<td>+0.3</td>
</tr>
<tr>
<td>Slope totally in filtering</td>
<td>1.037</td>
<td>1.511</td>
<td>-45.7</td>
</tr>
<tr>
<td>Slope partially in filtering</td>
<td>1.608</td>
<td>1.876</td>
<td>-16.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cases with surcharge (Clay – Janbu)</th>
<th>FS Macstars</th>
<th>FS Pendii</th>
<th>∆ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q = 0</td>
<td>2.233</td>
<td>2.263</td>
<td>-1.3</td>
</tr>
<tr>
<td>Q = 100 kPa</td>
<td>1.897</td>
<td>1.768</td>
<td>+5.8</td>
</tr>
<tr>
<td>Q = 500 kPa</td>
<td>1.363</td>
<td>1.127</td>
<td>+17.3</td>
</tr>
<tr>
<td>Q = 1000 kPa</td>
<td>1.144</td>
<td>0.891</td>
<td>+21.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cases with surcharge (Gravel – Janbu)</th>
<th>FS Macstars</th>
<th>FS Pendii</th>
<th>∆ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q = 0</td>
<td>2.671</td>
<td>2.907</td>
<td>-1.3</td>
</tr>
<tr>
<td>Q = 100 kPa</td>
<td>2.442</td>
<td>2.34</td>
<td>+4.2</td>
</tr>
<tr>
<td>Q = 500 kPa</td>
<td>1.835</td>
<td>1.531</td>
<td>+16.6</td>
</tr>
<tr>
<td>Q = 1000 kPa</td>
<td>1.539</td>
<td>1.214</td>
<td>+21.1</td>
</tr>
</tbody>
</table>

Tab. 2: comparison with the program Pangeo-Pendii

The difference between the two softwares are evident only for inclined water table (up to 50% when the water table is parallel to the slope profile), as Pangeo’s calculation hypothesis are not able to take into consideration the horizontal component of an inclined water table. Macstars, on the contrary, can correctly consider the hydrodynamic forces as confirmed by the comparison with the manual calculation (case 8 in tab.1)

As per the distributed load differences, this is due to the fact that in Macstars the distributed loads are transferred at the base of the loaded slices without any lateral diffusion, as it is considered in Pangeo.

In any case this difference seems to be evident only in case of presence of distributed loads much bigger than those normally found in the reality.

3 – COMPARISON BETWEEN SLOPE/W AND MACSTARS W

Slope/W is a slope stability program realised by Geo-Slope International, widely used for slope stability checks; the software allows to add reinforcements such as geosynthetics and tiebacks.
The comparison has been realised on a mixed structure made of three blocks, each one 5 m high, with 7 Terramesh elements and 3 geogrids Paralink 200 M.

Fig. 3: scheme used in the comparison with Slope/W

The results obtained are compared in the following graphs:

Fig. 4: comparison with program Slope/W. Minimum safety factors obtained with different limit equilibrium methods

Fig. 5: comparison with program Slope/W. Percentage movement of safety factor in respect to the reference value in Macstars-Bishop

Graphs show that in the global stability analysis the values obtained by the programs for the two analysed methods (Bishop and Janbu simplified) give very close results.

As for the local stability checks, where differences are more evident, this is due to the fact that they differently take into account the reinforcements interface shear forces, therefore if the failure surface intersects the reinforcements at the end of the anchoring stretch,
the software answer changes. This is due to the fact that Macstars uses a more conservative method not taking into account for the anchoring contribution the last 0.15 m of reinforcement, in order to consider the possible tolerances on the real length of the reinforcements at the jobsite.

As a further confirmation of that, notice that the differences on the third block, where the intersected reinforcement are less, the difference between the results is smaller.

4 – COMPARISON WITH TALREN SOFTWARE
The Talren software realised by Terrasol, allows the calculation of geotechnical structures with limit equilibrium method, for circular or non circular surfaces. The reinforcements which can be used by the program are: tie anchors, piles, micro piles and geosynthetics. The comparison between Talren and Macstars results has been realised on a mixed structure made of Terramesh System and geogrids Paralink 200M

And has given the following results [3]:

<table>
<thead>
<tr>
<th>Case</th>
<th>Critical surface – load combination</th>
<th>FS Macstars</th>
<th>FS Talren</th>
<th>Δ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Internal stability (combination A); circular failure surface at the toe</td>
<td>1.02</td>
<td>1.03</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Internal stability (combination A); circular failure surface at 2/3 from toe</td>
<td>1.24</td>
<td>1.24</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Internal stability (combination B); circular failure surface at the toe</td>
<td>1.27</td>
<td>1.24</td>
<td>+ 2</td>
</tr>
<tr>
<td>4</td>
<td>Internal stability (combination B); circular failure surface at 2/3 from toe</td>
<td>1.75</td>
<td>1.68</td>
<td>+ 4</td>
</tr>
<tr>
<td>5</td>
<td>Global stability (combination A);</td>
<td>1.21</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Global stability (combination B);</td>
<td>1.40</td>
<td>1.37</td>
<td>+ 2</td>
</tr>
</tbody>
</table>

Tab. 3: comparison with Talren program

As you can see the differences between the results are almost nil.

5 – COMPARISON WITH THEORETICAL EQUATIONS [4]
The analysis refers consisting of a long natural slope in a fissured overconsolidated clay inclined at 12° to the horizontal. The check is aimed to verify the correctness of Macstars in case of inclined water table: the values of the Macstars calculated SF are the same as the theoretical ones reported in [4].
9.4 ANALYSIS OF A PLANE TRANSLATIONAL SLIP

It is assumed that the potential failure surface is parallel to the surface of the slope and is at a depth that is small compared with the length of the slope. The slope can then be considered as being of infinite length, with end effects being ignored. The slope is inclined at angle $\beta$ to the horizontal and the depth of the failure plane is $z$, as shown in section in Fig. 9.7. The water table is taken to be parallel to the slope at a height of $mz$ ($0 < m < 1$) above the failure plane. Steady seepage is assumed to be taking place in a direction parallel to the slope. The forces on the sides of any vertical slice are equal and opposite and the stress conditions are the same at every point on the failure plane.

In terms of effective stress, the shear strength of the soil along the failure plane is

$$\tau_f = c' + (\sigma - u) \tan \phi'$$

and the factor of safety is

$$F = \frac{\tau_f}{\tau}$$

The expressions for $\sigma$, $\tau$ and $u$ are:

$$\sigma = \{(1 - m) \gamma + m\gamma_{sat}\} z \cos^2 \beta$$
$$\tau = \{(1 - m) \gamma + m\gamma_{sat}\} z \sin \beta \cos \beta$$
$$u = mz\gamma_w \cos^2 \beta$$
The following special cases are of interest. If \( c' = 0 \) and \( m = 0 \) (i.e. the soil between the surface and the failure plane is not fully saturated), then

\[
F = \frac{\tan \phi'}{\tan \beta}
\]  

(9.11)

If \( c' = 0 \) and \( m = 1 \) (i.e. the water table coincides with the surface of the slope), then:

\[
F = \frac{\gamma' \tan \phi'}{\gamma_{\text{sat}} \tan \beta}
\]  

(9.12)

It should be noted that when \( c' = 0 \) the factor of safety is independent of the depth \( z \). If \( c' \) is greater than zero, the factor of safety is a function of \( z \); in this case \( \beta \) may exceed \( \phi' \) provided \( z \) is less than a critical value.

For a total stress analysis the shear strength parameters \( c_u \) and \( \phi_u \) are used with a zero value of \( u \).

**Example 9.3**

A long natural slope in a fissured overconsolidated clay is inclined at \( 12^\circ \) to the horizontal. The water table is at the surface and seepage is roughly parallel to the slope. A slip has developed on a plane parallel to the surface at a depth of 5 m. The saturated unit weight of the clay is 20 kN/m\(^3\). The peak strength parameters are \( c' = 10 \) kN/m\(^2\) and \( \phi_{\text{max}}' = 26^\circ \); the residual strength parameters are \( c'_r = 0 \) and \( \phi'_r = 18^\circ \). Determine the factor of safety along the slip plane (a) in terms of the peak strength parameters, (b) in terms of the residual strength parameters.

With the water table at the surface \((m = 1)\), at any point on the slip plane,

\[
\sigma = \gamma_{\text{sat}} z \cos^2 \beta
\]

\[
= 20 \times 5 \times \cos^2 12^\circ = 95.5 \text{ kN/m}^2
\]

\[
\tau = \gamma_{\text{sat}} z \sin \beta \cos \beta
\]

\[
= 20 \times 5 \times \sin 12^\circ \times \cos 12^\circ = 20.3 \text{ kN/m}^2
\]

\[
u = \gamma_{w} z \cos^2 \beta
\]

\[
= 9.8 \times 5 \times \cos^2 12^\circ = 46.8 \text{ kN/m}^2
\]

Using the peak strength parameters,

\[
\tau_t = c' + (\sigma - u) \tan \phi_{\text{max}}'
\]

\[
= 10 + (95.5 - 46.8) = 33.8 \text{ kN/m}^2
\]

Then the factor of safety is given by

\[
F = \frac{\tau_t}{\tau} = \frac{33.8}{20.3} = 1.66
\]

Using the residual strength parameters, the factor of safety can be obtained from Equation 9.12:

\[
F = \frac{\gamma'}{\gamma_{\text{sat}}} \tan \phi'_r
\]

\[
= \frac{10.2}{20} \times \tan 18^\circ
\]

\[
= 0.78
\]
CONCLUSIONS
The obtained results put in evidence a full quantitative and qualitative agreement on the slope stability checks made with Macstars and those of the other softwares

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TECHNICAL NOTE 11: TYPICAL GEOTECHNICAL DATA TO MODEL GABIONS

GABION UNIT WEIGHT
The material used as Gabion filling should be hard, angular to round, durable, and have such quality that they shall not disintegrate on exposure to water or weathering during the life of the structure. The stone size should be 1 to 2 times bigger than the mesh opening.
The Gabion unit weight depends on the stone filling unit weight (variable between 23 kN/m³ of a sand stone to 29 kN/m³ of a basalt) and up on its porosity (variable from 30% to 40%).
Generally the gabion unit weight to be considered will be:

\[ \gamma = 17.5 \text{ kN/m}^3 \]

FRICITION ANGLE
The value of the gabion/stone filling friction angle depends upon different factors: the stone/rocks angularity (\(\phi\) increase with the angularity), the granulometry (\(\phi\) increase with the grain non uniformity) and the density (\(\phi\) increases with the density).
As a conservative value it is possible to assume:

\[ \phi = 40^\circ \]

COHESION
Tests performed on gabions allow determining the equivalent cohesion of the unit due to the mesh. The cohesion depends upon the amount of mesh per unit volume: it is greater for 0.5m high units compared to that of 1m high units, for gabions with diaphragms (i.e. Terramesh Systems), and for gabions having a stronger mesh (i.e. 8x10/2.7 compared to 6x8/2.2).
Knowing that Macstars considers for the Terramesh System (or gabions) only the contribution of the base mesh (not considering the diaphragm and lead contribution) it is possible to determine a value of a fictitious cohesion as follows:
The equivalent cohesion of a gabion \(c_g\) is in general expressed by the following empirical relationship:

\[ C_g = 0.03 \text{ Pu} - 0.05 \text{ [kg/cm}^2\]}

Where
Pu = mesh weight kg per m³ of gabion

In case of Terramesh System the Pu value (excluding the base mesh that is considered by Macstars for its tensile resistance) is 5.9 kg/m³ and therefore we obtain:

\[ C_g = 0.03 \times 5.9 - 0.05 = 0.127 \text{ kg/cm}^2 \]

Therefore the following value can be conservatively assumed:

\[ c_g = 12.5 \text{ kN/m}^2 \]
TECHNICAL NOTE 12: HOW MACSTARS TAKES INTO ACCOUNT THE CONTRIBUTION FROM THE WRAPPING

The length of the reinforcing element within the unstable soil block is divided into segments and for each segment the value of the ultimate tangential stress ($\tau_u$) is calculated by using the following equation:

$$\tau_u = f \sigma_v$$

where:

- $f$ = the total friction coefficient of the reinforcing element on the upper and lower materials in the segment considered, which can be reinforcement over reinforcement or reinforcement over soil;
- $\sigma_v$ = vertical stress acting on the segment considered.

The integration of the ultimate tangential stress provides the value of the ultimate internal pullout stress. In case of reinforcing elements on the facing, the resistance contribution due to the wrapped length of the reinforcing unit must be added. This contribution ($F_0$) can be calculated by the sum of two contributions:

$$F_0 = F_1 + F_2$$

Where $F_1$ is the contribution generating the pullout force on the tail (the 0.65 m horizontal stretch), whereas $F_2$ is the additional contribution which takes into consideration the stress acting on the sloped facing.

$F_1$ is calculated by using a procedure similar to the one used for the external pullout (integration of the ultimate tangential stress from 0 to $L=0.65$), taking into account the soil-reinforcement or the reinforcement-reinforcement friction, depending which side of the reinforcement's wrapping tail is considered (side up: reinforcement-reinforcement; side down: soil-reinforcement).

$$F_1 = \int_0^L \sigma_v(x) \cdot \mu \cdot \tan \beta \cdot dx + \int_0^L \sigma_v(x) \cdot \eta \cdot dx$$

where:

- $\mu$ = soil-reinforcement interaction (pullout) factor
- $\eta$ = reinforcement-reinforcement friction coefficient
- $\sigma_v = \gamma \cdot x \cdot \tan \beta$ = real vertical stress acting onto the tail (i.e. given only by the contribution of the soil column above the considered stretch $dx$ of reinforcement)

$F_2$ is calculated assuming that the facing has a semi-circular configuration as per the following, conservative model:
Where:
\[ p = \frac{F_1}{R} = \text{soil pressure on the facing, due to the anchoring force } F_1 \text{ of the wrapping horizontal tail. It has to be noted that this pressure is NOT the vertical pressure at the considered } Y \text{ level, but has a much smaller (and more realistic) value;} \]
\[ 2R = \text{diameter of the virtual circumference, equal to vertical spacing between reinforcements} \]